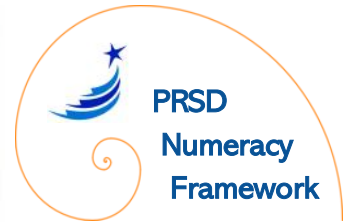
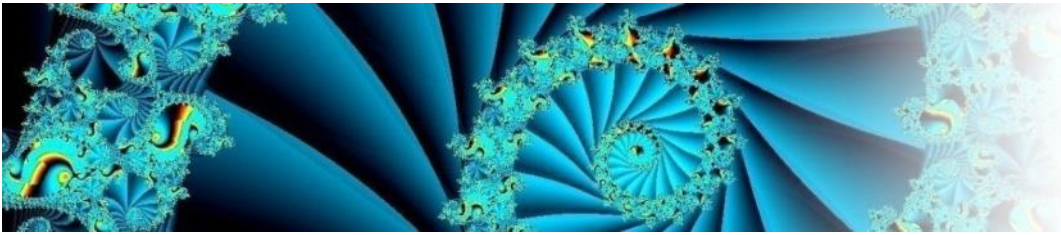




Peace River School Division

Learning Together - Success for All

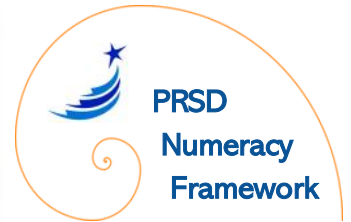
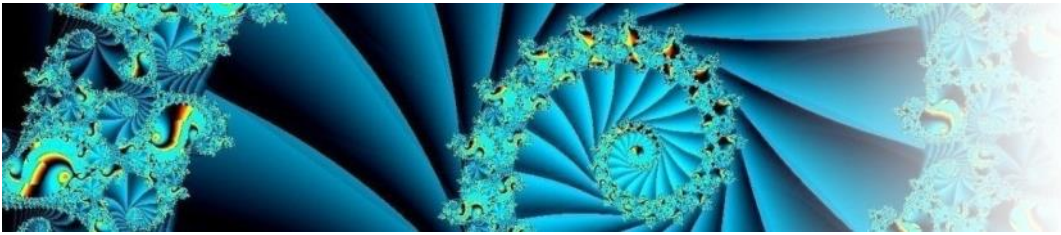
Numeracy Framework



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What is Numeracy?

Alberta Education (2016) defines numeracy as the ability, confidence and willingness to engage with quantitative and spatial information to make informed decisions in all aspects of daily living.

Quantitative information can be measured and expressed as an amount. It includes numbers, patterns, statistics, and probability.

Alberta Education, 2016

Spatial information is the physical location of objects or people or the relationship between objects or people. It includes measurements, location, direction, shape, and space.

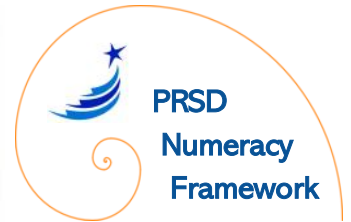
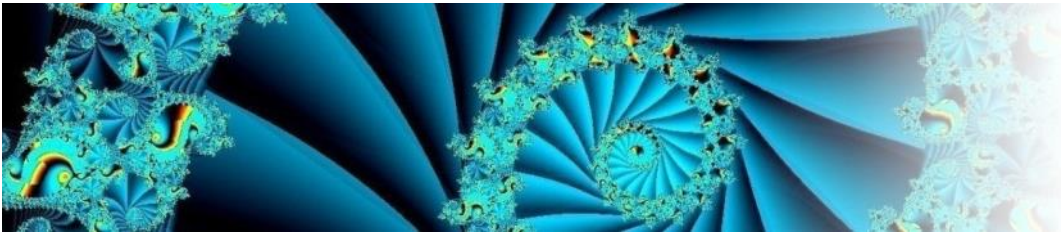
Alberta Education, 2016



A numerate individual has the confidence and awareness to know when and how to apply quantitative and spatial understandings at home, at school, at work, or in the community.

Alberta Education, 2016





Instruction and Assessment Schedules

Learning outcomes are organized for informing instructional planning, pacing, and assessment. Start where students are and move them forward in their journey.

The *essential outcomes* are from the number and patterns units.

- Long-range and unit plans are designed to ensure coverage of the essential outcomes
- Instruction, formative assessment, and interventions prioritize students' attainment of the essential outcomes

Instruction and Assessment Schedules (IAS)

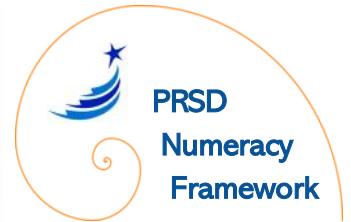
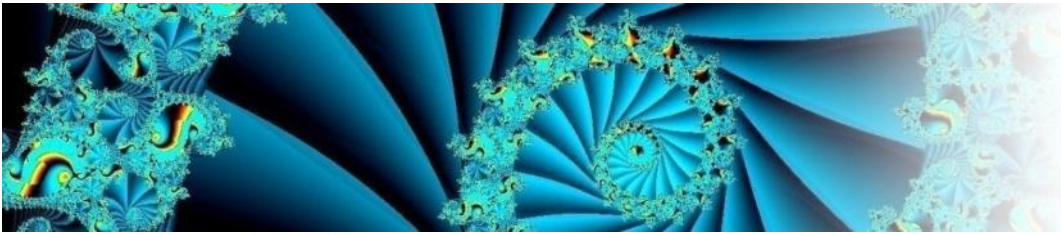
Prioritize and focus instruction and learning

Re-state or summarize individual or multiple curricular outcomes in student-friendly "I" statements

Assist in identifying which students require additional supports to learn key outcomes

IAS's for Grades K to 9 mathematics can be found in the [PRSD IAS's folder](#).





Numeracy Assessment Framework

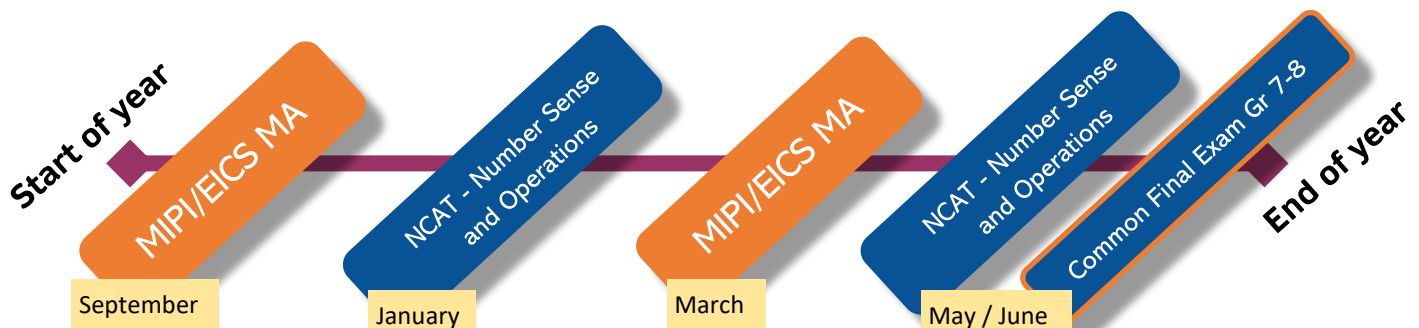
The NAF consists of two main assessment tools for PRSD math students:

Screeners and **Progress Monitoring** tools..

- ♦ Mathematics Intervention/Programming Instrument (MIPI)*
- ♦ Elk Island Catholic School Math Assessment (EICS MA)**

Numeracy Common Assessment Tool (NCAT)

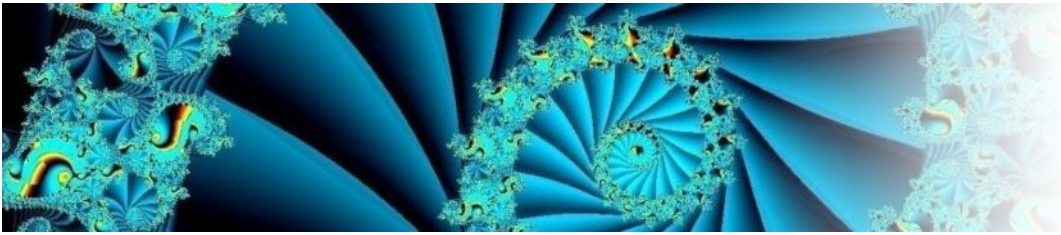
- **Screening**/benchmark tools: Assess concepts from the **previous year**
- EICS MA: all students in **Grades 1—7** at the start of the school year and **again** in March
- MIPI: all students in **Grades 8—10** at the start of the school year (optional in March).
- **Progress monitoring**/common assessment tool
- For all students **Gr. 1—9** after instruction of concepts in January and May/June.
- Assesses key grade-level concepts from the Number strand



* Used with permission from Edmonton Public Schools

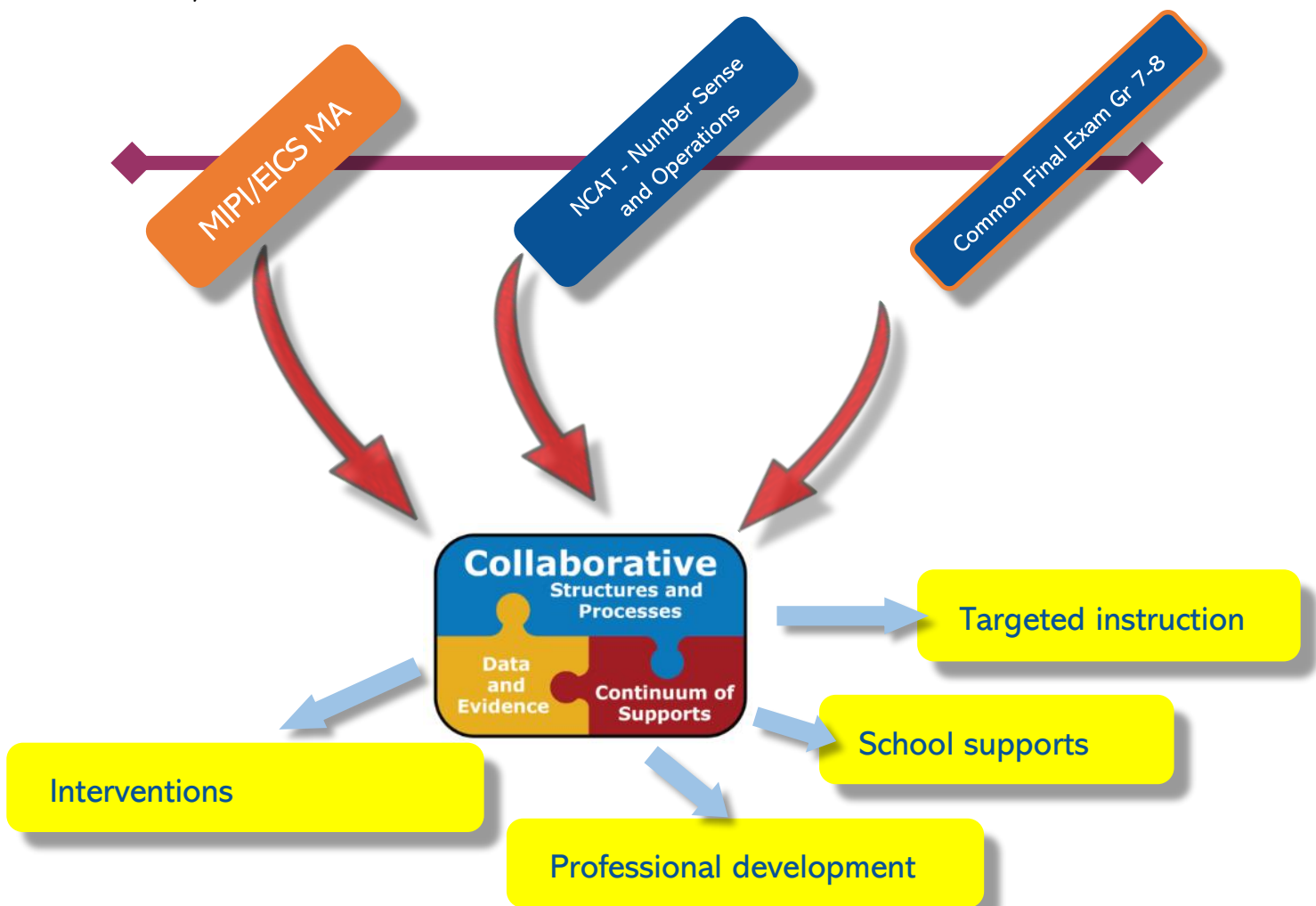
** Used with permission from Elk Island Catholic Schools

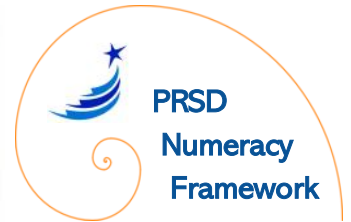
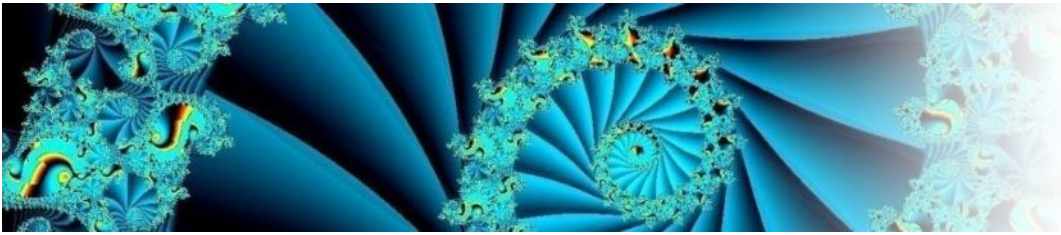




Numeracy Assessment Framework

Student achievement data from the MIPI/EICS MA and NCATs inform the **Collaborative Response** process at the classroom, school, and division levels.



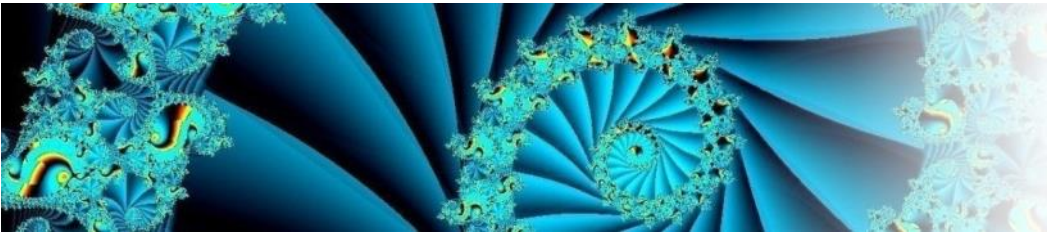


Numeracy Assessment Timeline ([Google Calendar Link](#))

Windows for implementing the Screeners and Progress Monitoring:

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
Gr. 1	EICS Math Assessment Screener (1st week of classes)				Number Sense Operations		EICS Math Assessment Screener (Same Assessment as the Fall)			Number Sense Operations
Gr. 2										
Gr. 3										
Gr. 4										
Gr. 5										
Gr. 6										
Gr. 7							EICS Math Assessment Screener (Same Assessment as the Fall)		Number Sense Operations	Common Final Exam
Gr. 8										
Gr. 9	MIPI						MIPI (Optional)			
Gr. 10										





Screening Tools (MIPI & EICS MA)

The benchmark/screening tools are to be used with all **Gr. 1 to 10** students at the **start of the year**. The EICS MA will be repeated again in March.

Spring reporting of the EICS MA is a requirement of Alberta Education and of our Partnership with EICS. Therefore, all schools must ensure the completion of the assessment and input the data in compliance with the schedule.

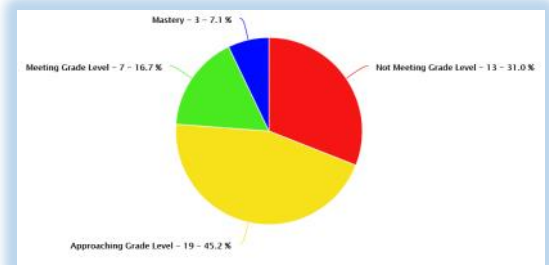
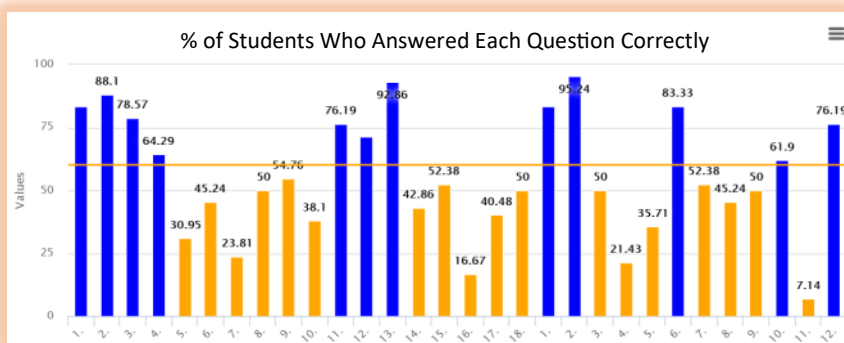
What are Screeners For?

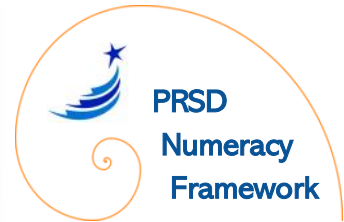
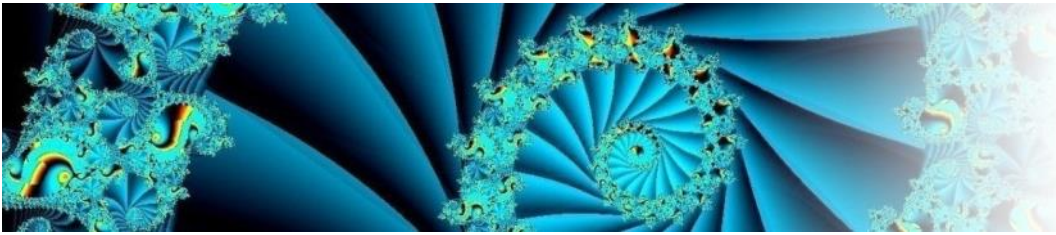
- Identifying generalities about class and student understanding of prerequisite skills
- Informing planning (e.g., How will gaps in prerequisite learning be addressed? How will individual students be supported?)
- Tracking class and grade performance year-to-year
- Identifying professional development needs

It looks at concepts from across the math curriculum of the **previous grade**, providing teachers with a gauge of **what was retained** and what will need additional review and support in the current year.



The MIPI and EICS MA provide teachers with various analyses through Dossier, including question-by-question, class, and individual student performance.





NCAT Curriculum Alignment

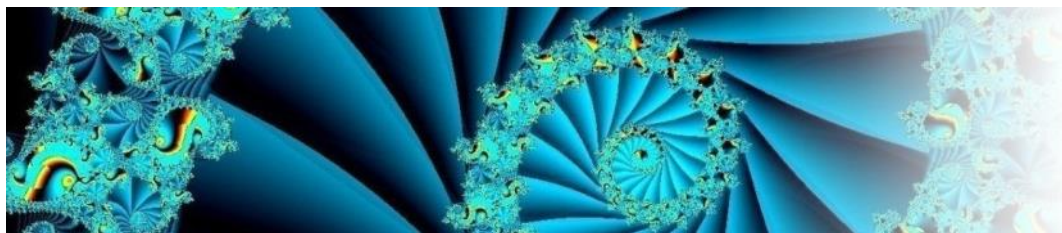
NCAT curricular
connections
Gr. 1-3

The NCAT assesses and monitors students' grasp of **key grade-level aspects** of the **Number** strand crucial to success **throughout the mathematics curriculum** as well as **subsequent grades**.

Each NCAT should be used after instruction of the applicable concepts.

	Grade 1	Grade 2	Grade 3
Number Sense	- Place Value: Whole numbers to 100 - Count forward by 1s, 5s and 10s (0–100); forward by 2s (0–20); and backward by 1s (20–0) - Compare and order numbers - Partition quantities (Half)	- Place Value: Whole numbers to 1000 - Count forward by 1s within 1000 - Skip count by 20, 25, and 50 from 0 - Skip count by 2 and 10 from any number - Compare and order numbers and fractions (including on a number line) - Even/Odd - Fractions (Part-Whole relationship) to 10ths - Determine value of a collection of bills or coins	- Place Value: Whole numbers to 100 000 - Round to a given place value within 100 000 - Compare, order numbers within 100 000 using $<$, $>$, or $=$ - Fractions to 12ths - Compare fractions with same denominator, numerator - Fractions on number line
Operations	- Add and subtract to 20 (including number lines) - Missing addends - Problem Solving	- Add and subtract to 100 - Problem Solving	- Add and subtract to 1000 - Multiply and divide within 100. - Problem Solving





NCAT Curriculum Alignment

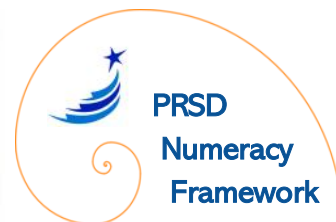
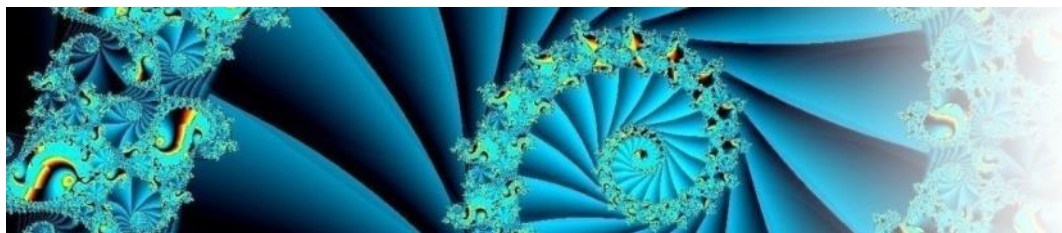
NCAT curricular
connections
Gr. 4-6

The NCAT assesses and monitors students' grasp of **key grade-level aspects** of the **Number** strand crucial to success **throughout the mathematics curriculum** as well as **subsequent grades**.

Each NCAT should be used **after instruction** of the applicable concepts.

	Grade 4	Grade 5	Grade 6
Number Sense	- Place Value: Whole numbers to 10 000 and decimals to hundredths - Fractions and parts of a whole (including equivalent fractions) - Convert between fractions, percents and decimals to hundredths - Compare and order numbers (including number lines) - Round numbers (including decimals) - Express monetary value in decimal notation - Prime and composite numbers	- Place Value: Whole numbers to ten million including decimals to thousandths - Convert between fractions and decimals to thousandths - Compare, order numbers and decimals to thousandths (including on a number line) - Compare fractions (including improper fractions) to benchmarks - Equivalent fractions and ratios	- Compare, order numbers integers (including on a number line) - Prime factorization, Factors and multiples - Exponents - Ratios and rates - Improper fractions and mixed numbers - Relating between fractions, decimals, percents
Operations	- Add and subtract to 10 000 - Add and subtract decimals to hundredths - Multiply and divide 2- or 3-digits by 1-digit - Problem Solving	- Add and subtract within one million and decimals to thousandths - Multiply 3- by 2-digits - Divide 3- by 1-digit, with and without remainders - Add and subtract fractions (Common denominators) - Problem Solving	- Addition of Integers - Addition and subtraction of fractions - Multiply and divide natural and decimal numbers - Order of operations (no exponents) - Problem solving using addition, subtraction, multiplication and division (including money)





NCAT Curriculum Alignment

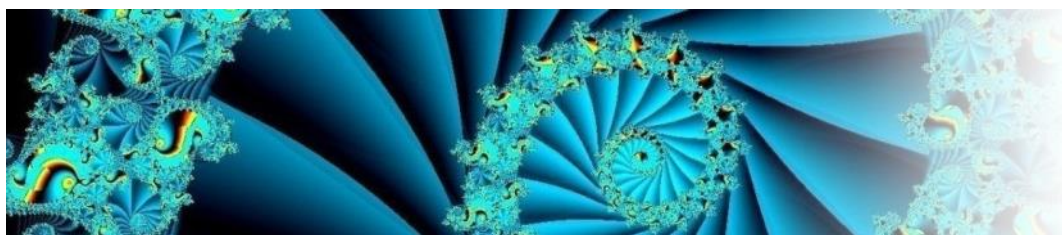
NCAT curricular
connections
Gr. 7-9

The NCAT assesses and monitors students' grasp of **key grade-level aspects** of the **Number** strand crucial to success **throughout the mathematics curriculum** as well as **subsequent grades**.

Each NCAT should be used after instruction of the applicable concepts.

	Grade 7	Grade 8	Grade 9
Number Sense	- Percent of a number - Converting decimals and fractions - Repeating and terminating decimals - Compare and order fractions and decimals (including on a number line)	- Percent (less than 1%, more than 100%) of a number - Squares and square roots - Estimating squares - Convert between decimals, percents, and fractions - Rate and ratios	- Square roots of rational numbers - Laws of exponents - Evaluating powers
Operations	- Add and subtract integers, fractions and decimals - Multiply and divide decimals - Problem solving	- Multiply and divide integers and fractions - Problem solving	- All operations with rational numbers - Order of operations - Problem solving





Instruction and Assessment Schedules (IAS) K- 9

Peace River School Division adopted a common scope and sequence for Grades 1 to 9 mathematics instruction.

PRSD: IASs Google Folder

Scope and sequence documents can be found in this Google Drive folder via your PRSD account.

Link to Google Drive

Why a Common Instructional Sequence?

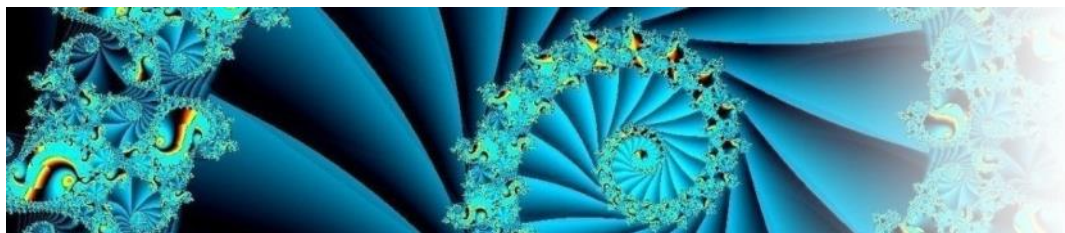
- More effective, timely delivery of professional development
- More effective collaboration between teachers, schools
- Support for students changing PRSD schools midyear
- Support for new teachers (first year, new to teaching math, new to teaching a particular grade of math)
- Universal use of pedagogically-sound progression
- Support for NCAT implementation and resulting interventions
- Instructional support for multi-grade and multi-age classrooms

Month	Week	Organizing Idea	Outcome Summary	P of S	"I can" Statement
September	1	Review and Screening (MIPI)			
	2	Number Concepts (Decimals and Percents)	Divisibility	N1	I apply divisibility rules to determine if a number is divisible by a given factor.
	3		Add, subtract Decimals	N2	I add and subtract decimals using a calculator
	4		Multiply, divide Decimals		I multiply and divide decimals using a calculator
October	5		Order of Operations		I can solve a given problem using operations on decimals
	6		Terminating and repeating decimals	N4	I distinguish between terminating and repeating decimals
			Convert fractions and decimals	N5	I convert between fractions and decimals
			Problems with Percent	N3	I can express a percent as a fraction and calculate a given percent of a number
			Compare and Order	N7	I apply the order of operations to solve problems 1-100%
			Review		Testing
	11	Number Concepts (Fractions)	Review: Convert between mixed and improper fractions		I convert concretely, pictorially, and symbolically between mixed numbers and improper fractions
	12		Simplest form and Compare	N5	I represent fractions in simplest form and compare and order fractions
			Common denominators		I convert fractions into common denominators
			Add, subtract like denominators		I add and subtract fractions with like denominators
			Add, subtract mixed		I add and subtract mixed numbers
			Add, subtract unlike denominators		I add and subtract fractions with unlike denominators
		Number Concepts (Integers)	Adding Integers		I add positive and negative integers
			Subtracting Integers		I subtract positive and negative integers
	19		Review		

Dates are guidelines to assist teachers in meeting pacing goals. Small variances between classes due to student needs are expected.

Designated weeks for completing the NCATs are "built in," but teachers can implement them earlier if desired.

Descriptive, student-friendly essential outcomes are available in a separate document.



Teaching for Conceptual Understanding

It is essential that teachers of mathematics in the elementary school understand the mathematics content they teach, know how students learn mathematics, and are able to use pedagogical strategies that support student learning of mathematics.

Gerretson et al., 2008, p. 303



A straight-to-algorithm approach forces most students to rely on rote memorization of abstract procedures. Dependent on these procedures, students are denied the opportunity to understand why the algorithms work, when to apply them, and how to recognize errors in their use.

Rather than simply learning to follow (by rote) a standard algorithm, students must be actively involved in reasoning and discussion ... with problems that are accessible to students through manipulation of objects, pictures, and other representations.

Milton et al., 2019, p. 33

A concrete-representative-abstract (CRA) approach to teaching mathematics fosters conceptual understanding beyond superficial procedures and should be considered a "best practice" in the elementary math class.

CONCRETE



REPRESENTATIVE

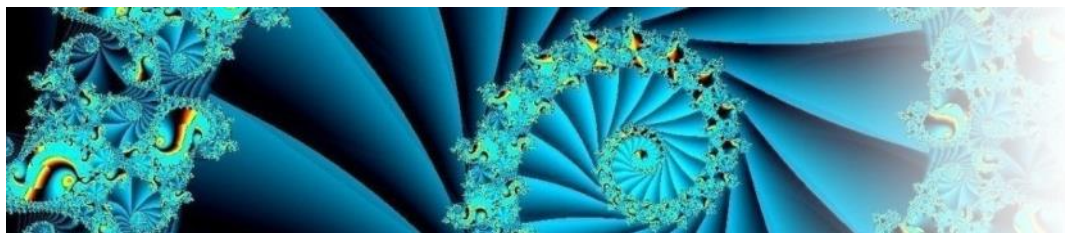


ABSTRACT



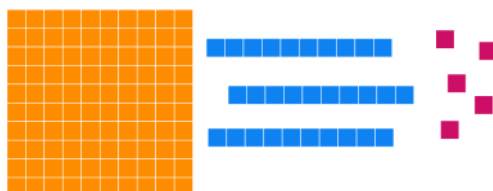
Peace River School Division
Learning Together - Success for All





CRA Progression

CONCRETE



Students first learn concepts through manipulation of concrete objects, forming conceptual understanding and internal representations.*

REPRESENTATIVE



Students make their own representations using pictures or diagrams, drawing from and further developing internalized understandings. *

ABSTRACT

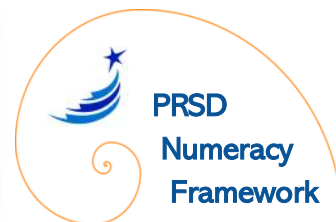
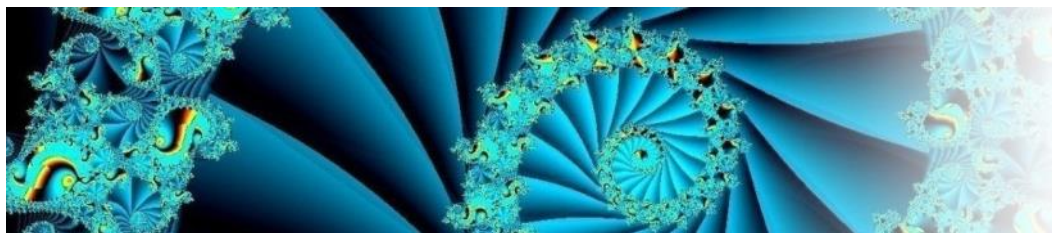
$$100 + 30 + 5$$

$$135$$

Students approach concepts using numbers only, associating previously formed representations with symbols. Instruction builds on conceptual understanding while developing procedural knowledge and fluency.*

* Milton et al., 2018





Fact Fluency, Not Memorization

Speed and memorization are two directions that we urgently need to move away from, not towards.

Boaler, 2019

Fact fluency is considered **one of the most important goals** of elementary mathematics instruction due, in part, to its frequent and foundational application throughout the curriculum leading into senior high school (Kling & Bay-Williams, 2015, p. 550).

Students unable to retrieve facts with automaticity are continually forced to “stop” and perform single-digit calculations **rather than focus on the higher concept at hand**. As the complexity of math increases, students lacking fact fluency are burdened with increasing cognitive loads, making simple yet essential tasks such as finding common denominators when adding and subtracting fractions extremely difficult (Baker & Cuevas, 2018).

Fluency does not — and should not — simply mean fast recall of rote memorized facts. A research-based approach is [on the next page](#).

Research tells us there is a distinct difference between memorizing and remembering. Memorization happens in a vacuum. It's like cooking on Teflon; it doesn't stick.

Newton, 2016, p. 14

Decades of drill and timed testing have failed our students, often leading to a lack of fluency and a negative disposition toward mathematics. Even in cases where students are able to successfully complete tasks, such as timed tests, one might question the value of such assessments. Does a perfect score on a timed test really tell us anything about that student's understanding?

Kling & Bay-Williams,
2015, p. 558

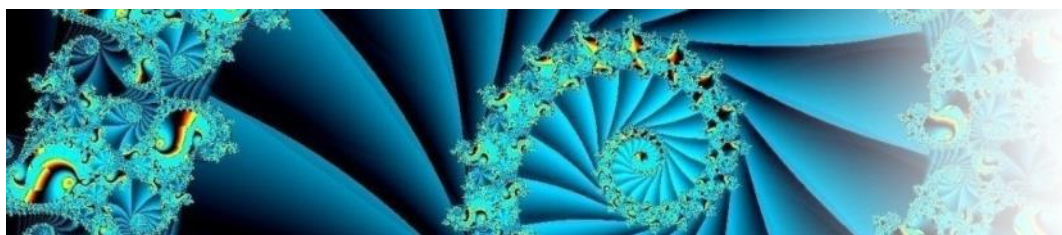
Research tells us the best mathematics classrooms are those in which students learn number facts and number sense through engaging activities that focus on mathematical understanding rather than rote memorization.

Boaler, 2019, Math 'Fluency' and the Curriculum, para. 2

Focus on memorizing individual combinations robs children of mathematical proficiency ... discourages looking for patterns and relationships ... deflects efforts to reason out answers ... and undermines interest in mathematics and confidence in mathematical ability.

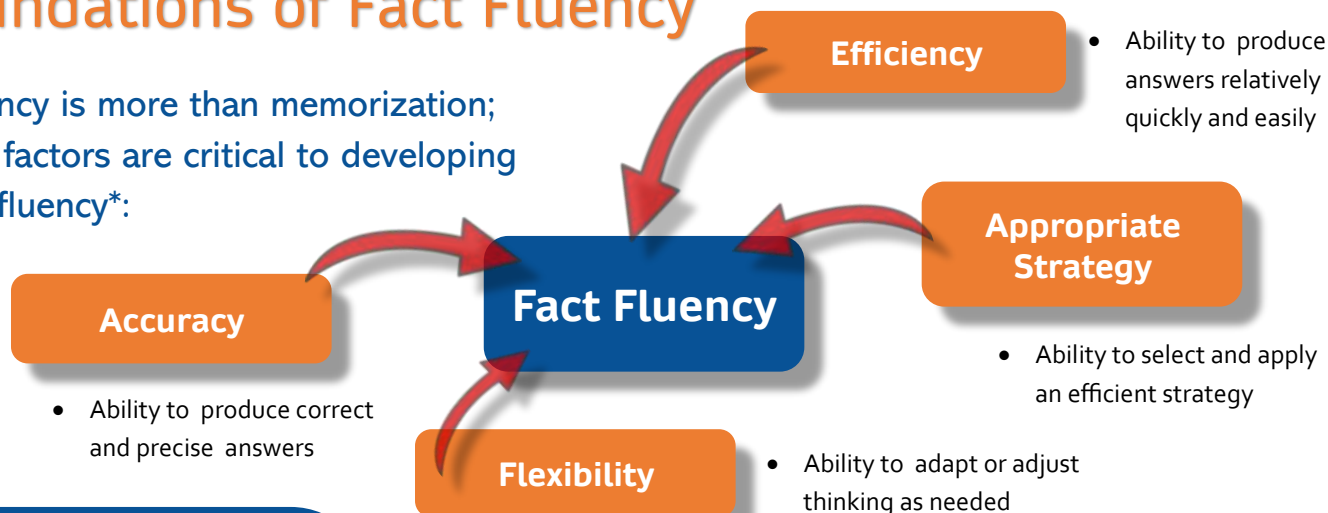
Baroody, 2006, p. 27





Foundations of Fact Fluency

Fluency is more than memorization; four factors are critical to developing fact fluency*:



Phases of Fluency Development*

1. **Counting**: counts concretely and mentally
2. **Deriving**: uses reasoning strategies based on known facts
3. **Mastery**: efficiently produces answers

Foundational Facts Must Precede Derived Facts*

+ 0, 1, 2 + 'Friends of Ten' + Doubles + Near doubles + 10 + __

Beginning facts → Final facts

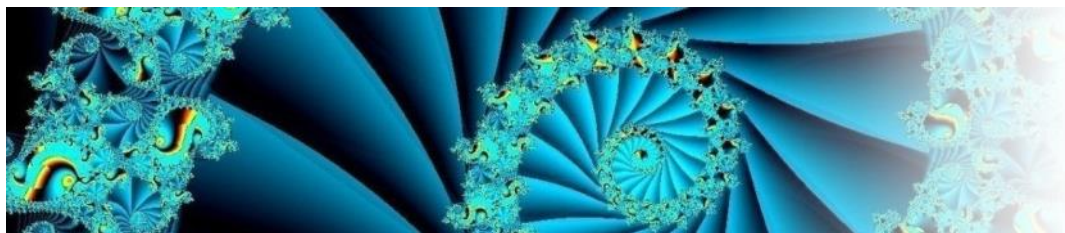
x 1, x 0 x 2, x 10, x 5 x Doubles x Squares x 4 x 3, x 6 x 9 x 7,

Timed Tests Do Not Assess Fluency*

- Timed drills do not allow teachers to see evidence of three of the four factors of fact fluency (above)
- Timed drills erroneously teach students that to be "good at math" means to be fast at math, and vice versa

*Baroody et al., 2016





Fact Fluency Via Conceptual Understanding

- Rote memorization
- Symbolic/abstract



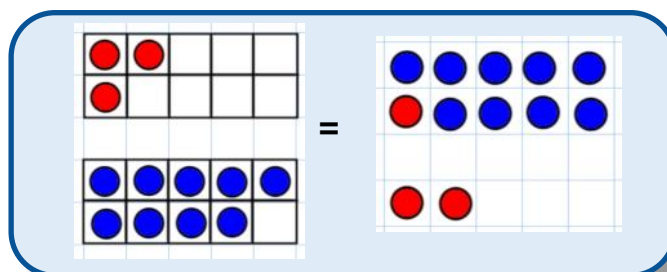
- Conceptual understanding
- Strategy development

- Conventional wisdom: Mastery grows out of rote memorization through repeated drill.

- Number sense approach: Fluency grows from conceptual understanding of the operation and development of strategies

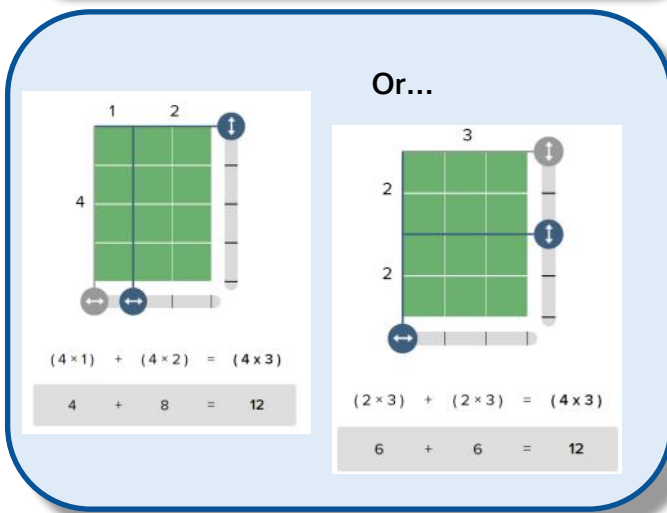
Example 1

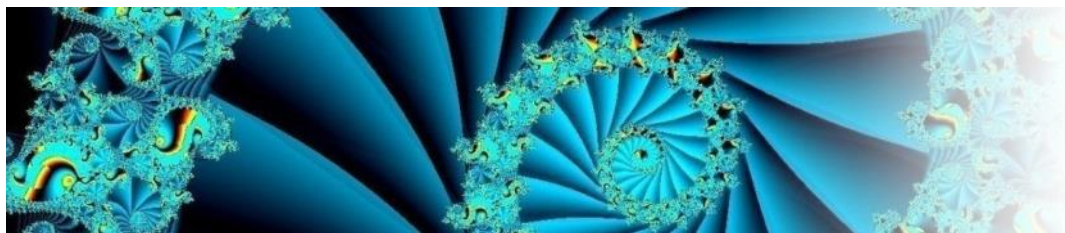
$$3 + 9 = 12$$



Example 2

$$4 \times 3 = 12$$





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